STUDENT LESSON

A17 – Quadratic Sorting Algorithms

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INTRODUCTION: In this lesson, you will learn about three sorting algorithms: bubble, selection, and insertion. You are responsible for knowing how they work, but you do not necessarily need to memorize and reproduce the code. After counting the number of steps of each algorithm, you will have a sense of the relative speeds of these three sorts.

The key topics for this lesson are:

A. Sorting Template Program
B. Bubble Sort
C. Selection Sort
D. Insertion Sort
E. Counting Steps - Quadratic Algorithms
F. Animated Sort Simulations
G. Sorting Objects

VOCABULARY:

BUBBLE SORT
INSERTION SORT
NONDECREASING ORDER
QUADRATIC
SELECTION SORT
STUB
SWAP

DISCUSSION:

A. Sorting Template Program

See *SortStep.java* and *SortsTemplate. java*

1. A program shell has been provided in the curriculum as *SortStep.java* (the main test method), and *SortsTemplate.java* (the sort class template).

2. The program asks the user to select a sorting algorithm, fills the array with an amount of data chosen by the user, calls the sorting algorithm, and prints out the data after it has been sorted.

3. At this point, each sorting algorithm has been left as a method stub. A stub is an incomplete routine that can be called but does not do anything yet. The stub will be filled in later as each algorithm is developed and understood.

4. Stub programming is a programming strategy. It allows for the coding and testing of algorithms in the context of a working program. As each sorting algorithm is completed, it can be added to the program shell and tested without having to complete the other sections.

5. This stepwise development of programs using stub programming will be used extensively in future lessons.

B. Bubble Sort

1. Bubble Sort is the simplest of the three sorting algorithms, and also the slowest. The Bubble Sort gets its name from the way that the largest items "bubble" to the top (end). The procedure goes like this.

   a. Move the largest remaining item in the current pass to the end of the data as follows. Starting with the first two items, swap them if necessary so that the larger item is after the smaller item. Now move over one position in the list and compare to the next item. Again swap the items if necessary.

   b. Remove the largest item most recently found from the data to be searched and perform another pass with this new data at step a.

   c. Repeat steps a and b above until the number of items to be searched is one.

   To see how Bubble Sort works, let's try an example:

Steps	**Data for pass**	**Sorted data**
Start pass 1: compare 4 & 1.	4 1 3 2	
4 > 1 so swapped, now compare 4 & 3.	1 4 3 2	
4 > 3 so swapped, now compare 4 & 2.	1 3 4 2	
4 > 2 so swapped, end of pass.	1 3 2 4	
Start pass 2: compare 1 & 3.	1 3 2	4
3 > 1 so no swap, now compare 3 & 2.	1 3 2	4
3 > 2 so swapped, end of pass.	1 2 3	4
Start pass 3: now compare 1 & 2.	1 2	3 4
2 > 1 so no swap.	1 2	3 4
Only one item in this pass so it is done.	1	2 3 4
Done.		1 2 3 4

2. The following program implements the Bubble Sort algorithm.

```
void bubbleSort(ArrayList <Integer> list){
  for (int outer = 0; outer < list.size() - 1; outer++){
    for (int inner = 0; inner < list.size()-outer-1;
inner++){
      if (list.get(inner) > list.get(inner + 1)){
        //swap list[inner] & list[inner+1]
        int temp = list.get(inner);
        list.set(inner, list.get(inner + 1));
        list.set(inner + 1, temp);
      }
    }
  }
}
```

3. Given a list of 6 values, the loop variables `outer` and `inner` will evaluate as follows.

When `outer` =	`inner` ranges from 0 to < (6 – `outer` – 1)
0	0 to 4
1	0 to 3
2	0 to 2
3	0 to 1
4	0 to 0

4. When `outer = 0`, then the `inner` loop will do 5 comparisons of pairs of values. As `inner` ranges from 0 to 4, it does the following comparisons:

`inner`	`if (list.get(inner) > list.get(inner + 1))`
0	`if list[0] > list[1]`
1	`if list[1] > list[2]`
…	…
4	`if list[4] > list[5]`

5. If `(list.get(inner) > list.get(inner + 1))` is **`true`**, then the values are out of order and a swap takes place. The swap takes three lines of code and uses a temporary variable `temp`.

6. After the first pass (`outer = 0`), the largest value will be in its final resting place (and may it rest in peace). When `outer = 1`, the `inner` loop only goes from 0 to 3 because a comparison between positions 4 and 5 is unnecessary. The `inner` loop is shrinking.

7. Because of the presence of duplicate values, this algorithm will result in a list sorted in non-decreasing order.

8. Here is a small list of data to test your understanding of Bubble Sort. Write in the correct sequence of integers after each advance of `outer`.

9.

outer	57	95	88	14	25	6
1	_____	_____	_____	_____	_____	_____
2	_____	_____	_____	_____	_____	_____
3	_____	_____	_____	_____	_____	_____
4	_____	_____	_____	_____	_____	_____
5	_____	_____	_____	_____	_____	_____

C. Selection Sort

1. The Selection Sort also makes several passes through the list. On each pass, it compares each remaining item to the smallest (or largest) item that has been found so far in the pass. In the example below, the Selection Sort method finds the smallest item on each pass. At the end of a pass, the smallest item found is swapped with the last remaining item for that pass. Thus, swapping only occurs once for each pass. Reducing the number of swaps makes the algorithm more efficient.

2. The logic of Selection Sort is similar to Bubble Sort except that fewer swaps are executed.

```
void selectionSort(ArrayList <Integer> list){
  int min, temp;

  for (int outer = 0; outer < list.size() - 1; outer++){
    min = outer;
    for (int inner = outer + 1; inner < list.size();
inner++){
      if (list.get(inner) < list.get(min)) {
        min = inner;  // a new smallest item is found
      }
    }
    //swap list[outer] & list[min]
    temp = list.get(outer);
    list.set(outer, list.get(min);
    list.set(min, temp);
  }
}
```

3. Again, assuming that we have a list of 6 numbers, the `outer` loop will range from 1 to 5. When `outer = 1`, we will look for the smallest value in the list and move it to the first position in the array.

4. However, when looking for the smallest value to place in position 1, we will not swap as we search through the list. The algorithm will check from indexes 1 to 5, keeping track of where the smallest value is found by saving the index of the smallest value in `min`. After we have found the location of the smallest value, we swap `list[outer]` and `list[min]`.

5. By keeping track of where the smallest value is located and swapping only once, we have a more efficient algorithm than Bubble Sort.

6. Here is a small list of numbers to test your understanding of Selection Sort. Fill in the correct numbers for each line after the execution of the `outer` loop.

`outer`	57	95	88	14	25	6
1	_____	_____	_____	_____	_____	_____
2	_____	_____	_____	_____	_____	_____
3	_____	_____	_____	_____	_____	_____

4 _____ _____ _____ _____ _____ _____

5 _____ _____ _____ _____ _____ _____

D. Insertion Sort

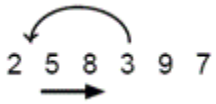
1. Insertion Sort takes advantage of the following fact.

   If A < B and B < C, then it follows that A < C. We can skip the comparison of A and C.

2. Consider the following partially sorted list of numbers.

   2 5 8 3 9 7

   The first three values of the list are sorted. The 4th value in the list, (3), needs to move back in the list between the 2 and 5.



   This involves two tasks, finding the correct insert point and a right shift of any values between the start and insertion point.

3. The code follows.

```
void insertionSort(ArrayList <Integer> list){
  for (int outer = 1; outer < list.size(); outer++){
    int position = outer;
    int key = list.get(position);

    // Shift larger values to the right
    while (position > 0 && list.get(position – 1) > key){
      list.set(position, list.get(position – 1));
      position--;
    }
    list.set(position,  key);
  }
}
```

4. By default, a list of one number is already sorted. Hence the `outer` loop skips position 0 and ranges from positions 1 to `list.size()`. For the sake of discussion, let us assume a list of 6 numbers.

5. For each pass of `outer`, the algorithm will determine two things concerning the value stored in `list[outer]`. First, it finds the location where `list[outer]` needs to be inserted in the list. Second, it does a right shift on sections of the array to make room for the inserted value if necessary.

6. Constructing the inner **`while`** loop is an appropriate place to apply DeMorgan's laws:

a. The inner **while** loop postcondition has two possibilities:
The value (`key`) is larger than its left neighbor.
The value (`key`) moves all the way back to position 0.

b. This can be summarized as:

```
(0 == position || list.get(position – 1) <= key)
```

c. If we negate the loop postcondition, we get the **while** loop boundary condition:

```
(0 != position && list.get(position – 1) > key)
```

d. This can also be rewritten as:

```
((position > 0) && (list.get(position – 1) > key))
```

7. The two halves of the boundary condition cover these situations:

`(position > 0)` –> we are still within the list, keep processing

`list[position - 1] > key` –> the value in `list[pos-1]` is larger than `key`, keep moving left (`position--`) to find the first value smaller than `key`.

8. The Insertion Sort algorithm is appropriate when a list of data is kept in sorted order with infrequent changes. If a new piece of data is added, probably at the end of the list, it will get quickly inserted into the correct position in the list. Many of the other values in the list do not move and the inner **`while`** loop will not be used except when inserting a new value into the list.

9. Here is the same list of six integers to practice Insertion Sort.

`outer`	57	95	88	14	25	6
2	_____	_____	_____	_____	_____	_____
3	_____	_____	_____	_____	_____	_____
4	_____	_____	_____	_____	_____	_____
5	_____	_____	_____	_____	_____	_____
6	_____	_____	_____	_____	_____	_____

E. Counting Steps - Quadratic Algorithms

1. These three sorting algorithms are categorized as quadratic sorts because the number of steps increases as a quadratic function of the size of the list.

2. It will be very helpful to study algorithms based on the number of steps they require to solve a problem. We will add code to the sorting template program and count the number of steps for each algorithm.

3. This will require the use of an instance variable - we'll call it `steps`. The steps variable will be maintained within the sorting class and be accessed through appropriate accessor and modifier methods. You will need to initialize `steps` to 0 at the appropriate spot in the main menu method.

4. For our purposes, we will only count comparisons of items in the list, and gets or sets within the list. These operations are typically the most expensive (time-wise) operations in a sort.

5. As you type in the sorting algorithms, add increment statements for the instance variable `steps`. For example, here is a revised version of the `bubbleSort` method:

```
public void bubbleSort(ArrayList <Comparable> list){
  steps = 0;
  for (int outer = 0; outer < list.size() - 1; outer++){
    for (int inner = 0; inner < list.size()-outer-1;
          inner++){
        steps += 3;//count one compare and 2 gets
        if (list.get(inner).compareTo(list.get(inner + 1)) >
0){
           steps += 4;//count 2 gets and 2 sets
           Comparable temp = list.get(inner);
           list.set(inner,list.get(inner + 1));
           list.set(inner + 1,temp);
        }
     }
  }
}
```

6. It is helpful to remember that a **`for`** statement is simply a compressed **`while`** statement. Each **`for`** loop has three parts: initialization, boundary check, and incrementation.

7. As you count the number of steps, an interesting result will show up in your data. As the size of the data set doubles, the number of steps executed increases by approximately four times, a “quadratic” rate.

8. Bubble Sort is an example of a quadratic algorithm in which the number of steps required increases at a quadratic rate as the size of the data set increases.

9. A quadratic equation in algebra is one with a squared term, like x^2. In our sorting example, as the size of the array increases to N, the number

of steps required for any of the quadratic sorts increases as a function of N^2.

F. Animated Sort Simulations

1. The web site titled "The Complete Collection of Algorithm Animations (CCAA)" located at http://www.cs.hope.edu/~alganim/ccaa/sorting.html, provides a wide variety of animated sorting simulations.

G. Sorting Objects

1. Notice that the sorts we developed above know how to compare Integers. Comparison is built into the Integer class. What if we wanted to write a sort that could work on Strings? You cannot use '<' on Strings. Remember you have to use the compareTo method.

2. To convert the BubbleSort, make the following changes that are highlighted in yellow.

```
void bubbleSort(ArrayList <String> list){
  for (int outer = 0; outer < list.length - 1; outer++){
    for (int inner = 0; inner < list.size()-outer-1;
inner++){
      if (list.get(inner).compareTo(list.get(inner + 1) >
0){
        //swap list[inner] & list[inner+1]
        String temp = list.get(inner);
        list.set(inner, list.get(inner + 1));
        list.set(inner + 1, temp);
      }
    }
  }
}
```

3. If I am able to sort my data, there must be an order defined for it. Classes that have an order should have a compareTo method. Java defines an `Interface`, Comparable, just for this purpose (see below for some information on Comparable). To make a BubbleSort that will work on any objects that implement Comparable, make the following changes, again highlighted in yellow.

```
void bubbleSort(ArrayList <Comparable> list){
  for (int outer = 0; outer < list.length - 1; outer++){
    for (int inner = 0; inner < list.size()-outer-1;
inner++){
      if (list.get(inner).compareTo(list.get(inner + 1) >
0){
        //swap list[inner] & list[inner+1]
        Comparable temp = list.get(inner);
        list.set(inner, list.get(inner + 1));
        list.set(inner + 1, temp);
      }
    }
  }
```

Now this method is quite reusable because we can use it to sort any Comparable object. The `compareTo` interface follows.

```
Interface java.lang.Comparable
int compareTo(Object other)
   // returns value < 0 if this is less than other
   // returns value = 0 if this is equal to other
   // returns value > 0 if this is greater than other
```

Remember to consider whether or not it makes sense to compare objects that you build. If it does, implement the Comparable Interface. It would also make sense to provide an equals method for your class.

SUMMARY/ REVIEW:

Sorting data is one of the best applications of computers and software. What takes hours or days by hand can be sorted in seconds or minutes by a computer. However, these quadratic algorithms have problems sorting large amounts of data. More efficient sorting algorithms will be covered in later lessons.

ASSIGNMENT:

Lab Assignment A17.1, *QuadSort*
Lab Assignment A17.1, *Starter Code – SortStep.java, SortsTemplate.java*
Worksheet A17.1, Bubble Sort
Worksheet A17.2, Selection Sort
Worksheet A17.3, Insertion Sort
Handout A17 - Answers, *Sorting Answers*